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STATISTICS AT MATHEMATICS LESSONS

Introduction

Around the world debate is still going on about the teaching of statistics and mathematics. Some even have doubts whether it is wise to include statistics into the teaching of mathematics, because they see a characteristic difference between them is that mathematics deals with thinking about certainty and statistics with thinking about uncertainty (Begg, et al. 2004). Watson (2006), however, claims the difference between mathematics and statistics is not so great if mathematics is taught as it should be, namely emphasising logical thinking instead of solving problems with learnt algorithms. We are namely daily confronted with statistical information, which we must know how to interpret and understand in the framework of a problem situation (Ben-Zvi & Garfield, 2004).

According to Tishkovskaya and Lanaster (2012) the use of relevant contexts and starting from students' own experience and understanding is highly recommended in building authentic understanding of statistics in the sense of developing statistical literacy. Also Sharma (2014) notes limited use of statistical thinking can be the consequence of the lack of different situations performed in the classroom and of inadequate curricular topics. The most often school curriculum envisages teaching statistics as a part of mathematics and this is why the discussed cases are more "mathematical", which certainly does not contribute to raising the level of statistical literacy.

Gal and Garfield (1997) claim a significant difference between mathematics and statistics primarily lies in the context. Stuart (2005) maintains that in solving statistical problems mathematical thinking dominates, and mentions the example of the limitations of mathematical thinking in the case of probability and distribution, because the traditional mathematical approach assumes that the probability theory is the foundation for statistics. He himself believes the opposite, that probability theory is not essential for the understanding of elementary methods of statistical inference, he does, however, not deny the importance of mathematics and mathematical thinking in statistics. Because strict probability theory is difficult, teachers tend to simplify the conceptual probability theory, so it becomes more easily understood for students; doing this, however, they often discourage them from understanding the problem in its original context.

Schield (1999) justifies the difference between mathematics and statistics in the light of the methods of inference and identifies mathematics, probability and statistics as examples of deductive reasoning, in which the argument can be correct or incorrect, and explains that

statistical literacy is an example of inductive and partly also of deductive reasoning. He explains inductive thinking as thinking that assesses arguments in a continuum (not within the context), where it is important that the stronger the argument the greater the likelihood is that decisions are accepted as correct. In addition to this, it is important for statistically literate students they prefer asking whether an assertion could be correct to asking whether it is correct.

Those who teach statistics should be aware they are not teaching an area of mathematics, as the teaching of statistics requires a completely unique method, which affects the development of statistical thinking of students (Forbes & Pfannkuch, 2009). Gal (2002) adds statistical thinking does not only depend on statistical knowledge, as the understanding of statistical information, its interpretation and response to it mainly depend on the general skills of literacy, mathematical knowledge, specific context and general knowledgeability.

In the updated syllabus for mathematics for Slovenian *gimnazija* (Žakelj et al., 2008), in which 560 periods are allotted to mathematics, the topics of data processing and normal distribution have been included. For the discussion of these topics 10 periods are envisaged, which is far from being sufficient. The recommendations state data processing is discussed in the first year of *gimnazija* already, primarily because of the coordination with the syllabus for mathematics in basic school, and normal distribution in year four. To avoid introducing some statistical concepts with the abstract probability calculus and for the learning of statistics to be dedicated to building statistical literacy or raising the level of it, also in *gimnazija* the introduction of the concepts of statistics must necessarily start from real life situations. In the syllabus it is emphasised students ought to know how to use the acquired knowledge about data processing in dealing with a statistical task taking account of all aspects of the corresponding problem situation. Also recommended is interdisciplinary cooperation with psychology, sociology, biology, sports education, physics and chemistry, and the use of information and communication technologies.

Students acquire the basic knowledge of data processing in the classroom, while more complex problems are solved in the framework of various activities, e.g. in the framework of interdisciplinary projects. Of course, the teacher who teaches mathematics is the one who is primarily responsible for the discussion of basic topics in data processing and statistics as well as for the discussion of more complex problems.

According to the recommendations in the syllabus for mathematics in Slovenian *gimnazija* (Žakelj et al., 2008) in mathematics classes students should acquire mathematical concepts and structures, develop critical thinking and mental processes, acquire formal knowledge and skills, and get to know the applicability of mathematics in everyday life. Especially in solving more complex problems students should use non-formal ways of thinking, be trained in precision and systematic approaches, and develop critical thinking. The acquired mathematical knowledge and experience that students acquire solving more complex realistic problems contributes to building mathematical competence, which is important for successful functioning in everyday life situations.

In mathematics classes the introduction into statistical topics is upgraded taking account of the knowledge students acquired in basic school. They know the basic statistical concepts, gather data, arrange and structure them, select the suitable presentation of data, read, produce, and interpret statistical representations of data, develop critical attitude towards interpretation of data as well as towards information in the media itself, analyse and produce a statistical task

and integrate crosscurricular themes into solving a statistical task, which should be taken from students' everyday life, and develop critical attitude towards interpretation of results. In teaching also making sense of the measures for centrality and dispersion is emphasised. Recommended is the discussion of realistic statistical problems for the solution of which not merely using basic statistical concepts and routine procedures are required, but creativity and ingenuity, and integration of different content areas.

Teaching mathematics and statistical literacy

In Slovenian *gimnazijas* poor care for statistics and statistical literacy shows in the fact that there is rather little verification of students' knowledge of statistics. After reviewing the *matura* examination sheets for mathematics at lower and at advanced level we notice in the last ten years statistics was not verified at all, as only in 2009 (RIC, 2009) a task was included in the examination sheet at basic level, with which the understanding of the concept of average value was verified. This indicates that in our country we have not yet become aware of the importance of statistical literacy.

Žakelj (2010) underlines in secondary school students ought to acquire statistical thinking and the skills of critical thinking to be able to read and interpret texts that include statistical data and their analysis. Using statistical methods, students should gain experience in solving statistical problems, and so in addressing statistical problems of everyday life promptly recognize how to interpret statistical information and use it during subsequent processes in decision-making. Watson (2006) maintains there are not many students who use school mathematics in further studies, there are, however, a lot of students who enter everyday environment where decisions are often concluded on the basis of data analysis. And if on the one hand school curriculum is intended for the preparation of students for everyday life and on the other hand mathematics curriculum provides logical basis and structure, which is the natural beginning of understanding, significance should by all means be given to statistical thinking, which is part of statistical literacy, as it is needed for students to become informed citizens. Students ought to be able to critically read a newspaper article, ask meaningful questions, and know how to critically judge given assertions, because the content of articles is not necessarily coordinated with school topics, but frequently refers to everyday life topics (Watson, 2006).

During schooling students also develop process knowledge, which can be used in other subject areas. In the framework of teaching mathematics they develop abstract thinking, understand the difference between formal mathematical inference and intuitive execution, in solving problems they use different strategies, develop effective reading strategies for further learning, communicate in mathematical language, plan, implement and present a research paper, ask key research questions, use information and communication technology, develop a critical attitude to information and apply mathematics in everyday life (Žakelj et al., 2008).

Due to the requirements of the syllabus and especially due to the requirements of the matura examination catalogue, however, the teaching of mathematics is frequently oriented into teaching and learning to the testing of knowledge. Moreover, in the syllabus guidelines for the development of statistical literacy have not been elaborated. This is why looking for an optimal approach to teaching statistics seems the more important, as it is not only about acquiring learning topics that are in the domain of teaching mathematics, but mainly about acquiring skills

that are extremely important for successful inclusion into everyday life. The topics of statistics are highly technically embedded into school mathematics, but in relation to the total number of hours of mathematics prescribed by the syllabus for *gimnazija*, a very small proportion of time is allotted for dealing with them. In dealing with statistical topics merely the knowledge and use of definitions, as well as procedural solving of statistical tasks with the application of procedures students have learnt by heart are mainly emphasised.

Similar things also occur in solving statistical problems. Here students typically follow the instruction and procedures in the way demonstrated by the teacher. Statistical problems are often artificial and not taken from students' real life. With this, real understanding of problems or problem situations, awareness of the meaning and effectiveness of the procedures of solving and the assessment of the correctness or incorrectness of solutions are therefore neglected. Such a way of teaching statistics does certainly not develop statistical literacy.

Ben-Zvi and Garfield (2004) underline statistics is taught very much "by bits" and that usually by teachers who neither have proper training nor are really enthusiastic about teaching statistics.

When introducing statistical concepts we first talk with students about their prior understanding of statistical concepts, especially about cases from everyday life (Felda, Lepičnik Vodopivec, & Bon Klanjšček, 2017). The use of relevant contexts and starting from the experience and understanding of students is extremely advisable for building genuine understanding of statistics in terms of developing statistical literacy. This is why the teaching of statistics should be oriented differently, which means a new approach is needed, with which every student will provide meaning and become aware of statistical concepts in the process of solving statistical problem situations or of solving statistical problems itself, which will contribute to raising the level of statistical literacy.

Because data and statistical thinking are more relevant to building statistical literacy than the mathematical context, statistical literacy is in a sense closer to art than to strict mathematics (Biggeri & Zulliani, 1999). Critical evaluation of statistical processing requires especially understanding of the whole statistical process from data gathering to analysis, verification of hypotheses, and evaluation of results (Ben-Zvi & Garfield, 2004; Pfannkuch & Wild, 2004).

In teaching and learning statistics the question must be asked which type of research would be the best and in addition to this also "out-of-school" statistics taken account of, so students are able to realise the practical meaning of it (Nasir, Hand, & Taylor, 2008). This is why in the syllabus for mathematics it is advisable to emphasise the significance of statistical literacy, as only by achieving a high enough level of statistical literacy students become active, curious, and critical citizens in modern society. Of course teachers, too, must understand statistics in its proper sense to be able to transmit it to students in the way to influence the development and ability of the use of statistical way of thinking, statistical literacy of teachers must therefore be promoted (Ridgway, Nicholson, & McCusker, 2007).

Raising the level of statistical literacy

Based on monitoring the reports of mathematics teachers in Slovenian *gimnazijas* on the teaching of statistics provided in meetings of study groups and in professional meetings, it was concluded that teachers mostly use such methods of teaching, where they first define a statistical

concept and explain the formula and then students consolidate these solving similar cases. In doing this, emphasis is on the computational aspect of solving statistical cases – they give priority to mathematical thinking. Unfortunately teachers often introduce individual statistical topics isolated from other topics, give priority to learning procedures by heart instead of understanding and integration of knowledge, and do not select tasks that are close to students. Teachers also do not develop effective strategies in solving problems and do not relate new knowledge to students' prior experience. Too little emphasis is put on encouraging students to critical thinking and critical interpretation of data – in general too little emphasis is put on the circumstances that are close to students. In solving statistical problems mathematical thinking is emphasised, while too little attention is paid to understanding the problem and to critical interpretation of results. In this way mathematics mostly becomes an obstacle to understanding the statistical problem, so this kind of teaching does certainly not contribute to developing statistical literacy.

To ensure raising the level of statistical literacy, certain changes must be introduced into the ways of teaching. As mentioned previously, it is very important for the development of statistical literacy students acquire statistical terminology and concepts in the framework of immediate experience in situations they are currently experiencing (tests, measurements, watching TV, etc.) or using contents they experienced in the past. Therefore, if examples from the lives of students or cases that students are currently experiencing are dealt with and in solving the cases students are directly applying statistics immediately experiencing it, this means they understand it and thus relate new definitions and concepts they are getting familiar with. Because students promptly experience the concepts, this means what they have extracted is directly logically related to the previously acquired structure of knowledge. This means thus students link everything they learnt before to experience – students' knowledge thus widens based on experience from problem situation and has clear meaning for them. It is extremely important they think of new definitions and concepts, thus integrating newly acquired learning into their existing knowledge.

For students learning statistics frequently depends to a large extent on the specific situation, they may orient their knowledge of statistics onto a concrete specific situation not being aware of the real meaning of statistics (Nasir, Hand, & Taylor, 2008). When examining students' knowledge, it is very difficult to verify whether students really master a definition or a concept. They may know definitions "technically", but we do not know whether they really understand the concepts; understanding concepts only becomes expressed through solving problems.

In solving realistic problems from everyday life students develop a critical attitude toward interpretation of data and develop the ability of critical evaluation of presentations and interpretations of statistical data in reports and in media, which certainly has impact on raising the level of statistical literacy.

School should be aware development of statistical literacy is crucial for the development of the next generation. In this both students and teachers are involved, who must permanently improve their knowledge and upgrade their statistical literacy. Through the integration of statistical concepts and statistical thinking, critically thinking and competent citizens, who will contribute to the development of society in general, should be educated from the very beginning of schooling all to the end of secondary school. Adequate development of didactic approaches as support to learning statistics is crucial for successful implementation of the essence of

statistical understanding of the school curriculum. This means universities should be involved in educating teachers in this sense and in taking care of their professional further education (UNECE, 2012).

Conclusion

For successful increase in the level of statistical literacy in the teaching of statistics it is therefore absolutely necessary teachers are well educated in this area. Regretfully, in our country the study of educational mathematics has been giving priority to probability calculus and less to statistical topics, where future teachers do not get enough opportunites either to understand statistical topics and to transfer them to their students, or to develop statistical inference, which serves giving meaning to statistical information and thus raising the level of statistical literacy. Undoubtedly, appropriate courses should be provided for teachers to be further educated in this area and to make up for the lost opportunities (Drobnič Vidic, 2008).

Although mathematics syllabus for Slovenian *gimnazija* (Žakelj et al., 2008) allots statistics only little space and merely recommends discussion of statistical problems from real world, statistical problems, precisely, should be given more emphasis. Exactly because the syllabus allocates so little time for the discussion of statistical topics, it is recommended in discussing them links are made to other subjects or suitable activities are carried out in the framework of project days, as in this way students could at all make meaning of statistical concepts and algorithms. The related problems require integration of different content areas and as such they are important for building and upgrading statistical literacy, which is, as we have previously underlined, a key factor of successful functioning in the society.

Realistic statistical problems are usually very complex problems that stem from concrete life situations and must be solved as the current situation requires it. This means one can detect a problem in her or his own way and by knowing concepts and algorithms solve it and verify whether the solution corresponds to the real situation.

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Abstract

There are some doubts about reliable learning statistics at mathematics lessons. Statistical reasoning actually differ from mathematical reasoning, but as the acquiring of statistical literacy has many similarities with the development of mathematical literacy and as mathematics and mathematical thinking are important in statistics, the doubts are dissipated. Nevertheless, the real weaknesses in learning and teaching statistics are inadequate approach to teaching, inadequate curricular topics and insufficient time for the acquisition and verification of statistical concepts.

Keywords: mathematics, statistics, realistic problems, statistical literacy