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CONSTRUCTIVISM IN THE TEACHING OF MATHEMATICS

Introduction

At the time of swift changes and of constant social development modernisation of school curriculum has been turning into permanent practice of developed school system around the globe. With its functioning school is expected to facilitate the development of those individual's competences that lead towards capacity for continuous learning. Summarising the most frequent goals of modernisation appearing today in developed school systems, the following could be mentioned: a shift from teaching to learning (Barr and Tagg, 1995), emphasis on the use of modern information technologies, qualification for work with diverse students (with diverse abilities, with special needs, multicultural diversity), the urgency of collaborating with other teachers and with parents, and being trained for reflection, investigation and evaluation of one's own work (see also Darling-Hammond, 1995). From the above it is evident that in reality it is not about entirely "new" roles, but about shifting the emphasis.

Modern curricular theories state (Schollaert, 2006) that modernisation of the curriculum or introduction of changes into the learning process can only be successful provided development and implementation take place jointly with teachers, with innovations being regularly tested in practice. With such an approach curriculum designers develop and implement conceptual solutions in collaboration with teachers, while introducing innovation into practice follows the model according to which development and implementation take place simultaneously.

In 2006 modernisation of curricula and syllabi started in Slovenia along the whole vertical (Smernice, 2007) from primary school to gymnasiums as an upgrade of the 1998 curricular reform. The modernisation followed the findings of domestic and international research, as well those based on monitoring classes, i.e. immediate school practice. Analyses had shown that it was necessary—inter alia—to modernise objectives and contents, as well as didactic approaches to learning and teaching.

While introducing change into our school system it became clear with the constantly evolving syllabi old methods of work (teaching computing proficiency) were vanishing and many known skills (teaching from the front) had become obsolete and inefficient. To attain the set goals in the area of fundamental, procedural, and problem knowledge, according to Gagne's taxonomy of knowledge, new opportunities arise that require new abilities and skills. The key solution lies in training, learning based on active involvement of participants of the learning process, which can be called learning through activities. Such learning is based on the

constructivist approach, experiential learning, problem learning, etc. (Rutar Ilc, 2002).

What modern school reforms around the world endeavour to accomplish in students is understandable, meaningful, and lasting knowledge. Constructivist theories of knowledge are based on the assumption individuals shape their knowledge by building their own knowledge (Plut-Pregelj, 2003); this is why advocates of the constructivist theory believe that constructivist theories of knowledge precisely should become the main starting point for the modernisation of the curriculum (Rutar Ilc, 2002).

The constructivist way of learning and teaching

First, the concepts of teaching and learning need to be defined. Teaching is understood mainly as transfer of knowledge and learning as assimilation or acquisition of knowledge. This is why rather than transfer of knowledge teaching ought to mean provision of conditions in which active approach is possible, which means discovering and building of knowledge and not merely absorbing it.

The question arises what learning is defined as active learning. It is a fact that all learning is active, as it involves certain mental effort, even though it is only about memorising facts. We could claim, therefore, all learning is active. Learning isolated information without forming new conceptual links does nevertheless not lead to lasting and quality knowledge. One should, likewise, not equate mental activities with activities in which students are using didactic materials. Learning and teaching must assure interaction between concrete and mental activity that leads to links. Based on the above we shall henceforth understand the term “active learning” is used to denote that mental activity that leads to links between mental and concrete activity (Žakelj, 2004).

A number of studies (Bransford, Brown, and Cocking, 2000) have shown the durability and usefulness of knowledge acquired in a so-called active way is greater than when knowledge is merely assumed, because the learners who have discovered certain mathematical concepts with active investigation, have used this knowledge in new and atypical situations, while the students who have just become acquainted with these concepts (learnt them by heart), have failed in new situations (Rutar Ilc, 2002).

Cognitive psychologists (Piaget, Vygotsky, Bruner, Bransford, and Marzano; as cited in Rutar Ilc, 2002) have emphasised the relevance of such activities in which students arrive at their own findings or build them by themselves, in the process of investigating and discovering, with deliberated teacher’s support.

With the so-called transmission approach, where it is mainly about transfer of ready-made knowledge, students are thus active to a certain extent, as they are listening to teacher’s explanation trying to comprehend it. Nevertheless, students are not active in most of the important phases of cognitive process, but just made acquainted with the findings that are the result of a cognitive path someone else has walked, while they are not taking it themselves (Rutar Ilc, 2002).

Active approach, which is based on one’s own discovery and building findings with the aid of various activities and mental processes and procedures that these activities stimulate is what—in a rather passive absorption of ready-made knowledge—allows internalisation of concepts, principles, and laws and with this also durability and transfer value of knowledge

(Rutar Ilc, 2002).

In today's school practice it happens all too often that mainly due to lack of time we teachers look for a way of teaching that will quickly bring us to the set goal. Sometimes the path may really be shorter, especially when we are using passive teaching methods such as explanation, demonstration, discussion, etc. — not being aware this kind of student's passive acquisition of learning matter leads to acquisition of and managing lower taxonomy levels of knowledge that do not allow the student to develop adequate mental processes and skills that lead to applicable knowledge.

Modern strategies of learning and teaching have brought great changes into the teaching of mathematics and in consequence also into examination and assessment of knowledge. The goals of mathematical education, which used to be oriented on acquisition of concrete contents and predominantly procedural knowledge and skills, have nowadays been increasingly complemented with conceptual and process knowledge or knowledge oriented into finding ways and strategies of solving problems that are also transferrable to other subject and extracurricular areas (Cotič, 2010).

In today's world the relevance of procedural knowledge has been reduced, while at the same time the need for problem-solving knowledge and skills has increased. Increasing importance of complex knowledge has been emphasised that ranges from basic reading skills and computing to awareness of complex problems and solving them. There have been changes in the way of thinking what knowledge really is – from understanding knowledge as unambiguous and unchangeable to complex and dynamic. The starting point of studying kinds and aspects of knowledge is represented by theories and classifications of knowledge (Žakelj, 2003). It is crucial that we—primary school teachers, mathematics teachers, and didacticians of mathematics—are all aware of different aspects, kinds, and levels of knowledge, that we are able to judge which should be given precedence in a given situation, and that we know in what ways to introduce, discuss, consolidate, and eventually also examine and assess them in mathematics classes (Cotič, 2010).

It is assumed our society of the future is going to be “knowledge society”, the main condition and path to this goal is therefore a quality education system based on well-trained competent teachers (Peklaj, 2007; Valenčič Zuljan, Krištof, Vogrinc, Brank, Pohar, and Bizjak, 2005; Veenman, 1984).

Teachers always strive to teach well; to reach this goal, however, power and energy as well as the ability of influencing students are needed, which they attain with well deliberated learning and teaching methods (Rutar Ilc, 2002).

The precondition for quality education in the so-called excellent school is therefore the teacher and his qualification to attain the educational goals in the work with which he can enable students to develop their potential, inclusion into society and to use their knowledge and competences in diverse areas to become an agent of the development of the society. In his work the teacher is thus expected to be able to promote the overall development of students at cognitive, emotional-motivational, and social levels. This is why the teacher must be trained well and precisely because of this his competences are in the forefront, as the teacher exactly is the one who is expected to be able to offer the student these competences for survival in the increasingly complex society (Peklaj, Kalin, Pečjak, Puklek Levpušček, Valenčič Zuljan, and Ajdišek, 2009).

In the 90s of the 20th century several organisations endeavoured to define the concept of competences wishing to determine the most important competences as indicators of development relevant for the whole human society. Competences were thus defined (Rychen and Salganik, 2003) as “the ability to attain complex demand in a certain context with the support of both cognitive and non-cognitive aspects of functioning” (Peklaj et al., 2009). This means a developed competence must be applicable in a variety of situations. What follows from this is that competences include cognitive level (the ability of complex thinking and problem solving as well as knowledge in a certain area), emotional-motivational level (attitudes, values, readiness for activity), and behavioural level (the ability to adequately activate, coordinate, and exploit one’s potentials in complex situations) (Peklaj et al., 2009).

The organisations further focused on the concept of key competences and that with the definition that those competences are selected as key competences that contribute to highly valued achievements at individual and social level in the sense of successful life and well functioning society, as with them relevant, complex requirements and challenges are to be achieved in a wide spectre of environments assessed as being important for everyone. In this way three basic groups of key competences were elaborated:

- functioning in heterogeneous groups,
- acting autonomously, and
- using tools interactively (Peklaj et al., 2009).

Quality and efficient education system implemented by well-trained teachers, however, certainly remains the basic condition for successful development and acquisition of any competences (Peklaj et al., 2009).

According to Leonard cognitivism exactly is one of the basic theories of learning where it is about believing human thinking and learning are similar to that of a computer — a robot, to computer processing of information. In education cognitivist orients into transformation of knowledge from real world via teacher’s intervention to learner. Success has been attained if at the end of the lesson the student has met the objectives set by the teacher. The difficulty that arises is how to determine whether the student has really attained the goal. Cognitivism is completely oriented into inner mental process systems or into learning schemes in the context how the brain receives, internalises, and retrieves information. In constructivism the learner builds his meaning, image based on new knowledge that helps him construct new knowledge (Leonard, 2002).

Constructivism is the belief students have some prior knowledge and experiences on which their assumptions (hypotheses) are based and that serve as the basis for shaping the context, the base for solving the concrete problem triggered by the teacher. Constructivism is a student-centred educational paradigm, in which the content is built by the learner in a team-based group in the form of collaborative learning and in the framework of the constructivist learning environment open for construction. The theory of constructivism is focussed on learner’s thinking, on his learning activities. In the active learning paradigm of constructivism teacher is no longer the primary medium and the only channel for providing knowledge. Entire knowledge goes first directly to learner himself. The teacher is the catalyst, the advisor, and the leader of the programme of implementation of projects for solving a concrete problem and not an obstacle between the learner and the content. With constructivism learning investigation takes place, research, discovering of autonomous self-motivating learner, which is the critical momentum

important for successful implementation of the learning process (Leonard, 2002).

Barica Marentič Požarnik (2000) defines constructivism as the psychological orientation that emphasises the relevance of man's internal, especially cognitive processes in learning and achieving deeper understanding. She says about constructivists they go a step further, as they believe knowledge in ready-made form can neither be given to another person nor received from someone; everyone must build it anew with their own mental activity. Adherents of constructivism thus substantiate knowledge is not received from the outside but built by ourselves with our own activity in the process of giving meaning to our experiences. What follows from this is that knowledge is not something that exists objectively independently by the one who cognises, but a subjective construct that every learner creates in the process of providing meaning to their experiences (Marentič Požarnik, 2000).

Požarnik conceives constructivism as a holistic view of learning, which is not only cognitive activity of the human, but necessarily unites the emotional, motivational and social dimension of the individual. In this way the internal motivation of a person for a particular subject or problem that is being built – constructed in the process of solving reasonable problems is encouraged and thus a different quality of learning than with predominantly external motivation (Marentič Požarnik, 2004).

Cognitive development theory emphasises the roles of both the teacher and the student in the learning process, as they each contribute their parts. In this theory the essence of teaching is the interactive and dialectic teacher–learner relationship; it is about the student being active in the learning process. Teacher's task is only to encourage and develop students' intellectual and social development.

The emphasis in this theory is based on the premise that in teaching it is necessary to take account of the learner, whose development takes place in four different stages, namely: the sensorimotoric stage, the pre-operational, the concrete operational, and the formal operational stage (Piaget, 1974, as cited in Sprinthall, Sprinthall, and Oja, 1994).

This developmental theory of teaching is based on the dialectics of balance and imbalance and on the need of students for assimilation and accommodation, which assumes that if the balance between old and new knowledge has been destroyed, emergence of new knowledge can be expected.

In teaching teacher's understanding of students' cognitive development and the use of teaching strategies, which depend on teacher's stage of cognitive development, play an important role. Teachers with highly developed concepts are thus better able to adapt to situations, are better at selecting highly structured material for students at a higher developmental level and more concrete material for those at a lower level of mental development, and they adapt their approaches accordingly. Conversely, teachers with less developed concepts use proven methods for all students, they do not use flexible teaching, independent learning, or learning in groups.

With cognitive developmental theory the teacher and the student are necessarily active and interdependent in the teaching process. In this theory it is about mental processes and development stages in the learning process that clearly delineate the roles of the teacher and of the learner in the process of constructing new knowledge (Zabukovec, 1997).

Today school requires learning and teaching that is adapted to the interest and abilities of learners, that takes account of their acquired experience, and that allows them active

participation in the learning process. Due to these requirements it is necessary for teaching to allow students learning new matter with practical activities. Through activities students develop certain skills and procedures that—based on these practical activities—allow them to acquire mental activities.

The performance of such teaching is made possible by various up to date didactic strategies called open teaching (Ivanuš Grmek and Hus, 2006). Characteristics of open teaching are manifested in a variety of didactic approaches: in teaching oriented into research, project-work, problem solving, team-work, work-oriented learning, or experiential learning.

Didactic approaches play an important role in understanding new concepts and facts and in the acquisition of new knowledge (Petek, 2005).

For the attainment of the required goals of modern schooling it is thus necessary to complement the transmission (behaviourist) model of teaching with modern cognitive-constructivist or transformational model of teaching. This requires problem-oriented methods and approaches, where the teacher pays attention not only to the quantity but also to the quality of learners' prior knowledge. The teacher purposefully exploits students' experience, attitudes, and views, confronts them with the imperfection and with the conflicting nature and with adapted support assists students in the reconstruction of knowledge. In the cognitive-constructivist model the importance of students' activities at all stages of learning, of collaboration, of exchanging experience and views among students, and of planned acquisition of learning skills is emphasised. In such learning and teaching the student gradually assumes increasingly larger part of responsibility for the process of the acquisition of knowledge and of personal development and gets trained for lifelong learning (Valenčič Zuljan, 2002).

With such teaching teacher's role becomes more demanding, as teaching does not follow a predefined scenario. Much remains unforeseen, a lot depends on learners, the teacher must therefore be adaptable, but nevertheless cautious not to miss the set learning objectives.

Marentič Požarnik points out the teacher who wishes to teach in this way must have the following competences:

- to be able to elicit students' existing ideas and experience;
- giving students the task of solving meaningful complex problems;
- guiding students to conduct independent thinking and find living links;
 - modelling mental processes in front of students, in this way teaching them to “think”, leading mental dialogue about tasks in front of them;
- encouraging students to explain, justify, and infer conclusions;
- establishing relaxed classroom atmosphere;
 - using a variety of examination strategies and allowing students to participate in defining examination and assessment criteria (Marentič Požarnik, 2008).

Today teacher education science highlights the inseparable interlacement of theory and practice in the training of teachers as “reflective professionals” able to cope with new challenges and to successfully face the complexity and uncertainty of their profession. It is therefore not by accident that the EU has earmarked considerable amounts of money for financing projects targeted at strengthening partnerships and collaboration between higher education and schools, ensuring thus higher quality attainment of competences teachers of the future need for more successful professional functioning. The teacher as the “reflective professional” is in the focus

of attention in planning changes in education. For the implementation thereof, however, both systemic-organisational and content-executive aspects of teacher education are important (Marentič Požarnik, 2004).

In theory and practice constructivism is expressed in a number of directions. Common to all is the educational philosophy that the student is the chief constructor of his own knowledge. Knowledge is therefore the product of one's own mental processing, which because of specificities of the individual is expressed in individual varieties.

In the planning, organisation, and implementation of learning characteristics of teaching are important to constructivist teachers such as emphasis on learning rather than on teaching; encouraging student's curiosity and initiative; learning as a process rather than as an event; experiential learning under the lens of teacher's critical judgement; focused teacher's attention to student's mental habits, understanding, and application of cognitive concepts; encouraging cooperative learning; exposing students in realistic events; examination of student's attitudes and views (Krapše, 2002).

Any individual with adequate university education can become a teacher, but not everyone can be a good teacher. In today's time requirements concerning teaching have quickly been changing and they position the teacher into a role in which passive "delivery" of information to students is not enough or is even undesired, because such method of teaching is outdated and does not yield desired results that are more or less measurable with external tests or national and international studies of knowledge. How to be or become a good teacher is a question constantly topical in education processes.

Since 1998, when the Syllabus for mathematics in the nine-year basic school (Učni načrt za matematiko v devetletni osnovni šoli) was adopted changes have been introduced into the teaching of mathematics. In contrast to emphasising merely automation of arithmetic facts problem-solving approach mainly began to be highlighted. In spite of this, even today a lot of attention is still dedicated to results rather than processes that students use in solving problem-based tasks (Felda, 2011).

In the modern world there are countless research and studies on how to improve teaching methods. All or most of these lead to the same solution, namely to the shift from teaching to learning. This means that the teacher is no longer at the centre of the educational process; the learner is gaining an increasingly greater role. Nevertheless, the teacher plays a key role in this process, since he appropriately prepares the material for the discussion, provides orientation and indirectly guides learners through the learning process, and identifies the achievement of the previously set learning goals and "corrects" pupil's wrong ideas. The teacher can, however, only carry out all this work using modern teaching methods, where students' own activity is the main guideline.

The solution to the problems of many an education system is thus quite obvious – learning of learning as defined as the ability to learn and persist in learning, including with efficient management of time and information, individually and in groups. In this learning it is about the awareness of one's own learning process and needs, recognition of available opportunities, and the ability of overcoming obstacles for successful learning. It is about absorption, processing, and acquisition of new knowledge and skills and finding and using advice. By learning to learn students upgrade their prior experience with learning and life experience in different circumstances (Recommendation of the European Parliament and of the Council of the EU,

2006).

Conclusion

In spite of all the positive effects of the constructivist way of teaching in mathematics, what all too often happens in school practice when schools notice a change has been introduced is that teachers relapse into the old ways of doing things. Schools are expected to be learning communities, which means they must take responsibility for their own learning. To be able to do this, they must develop the ability to respond to the challenges of the changing world while at the same time preserving the awareness about their moral responsibility towards the learners they have been trusted with.

If we wish the constructivist model of teaching to be transferred into school practice, it must also be introduced into initial training of teachers at universities. This means teachers-to-be must themselves get familiar with such a model of learning and teaching, as in addition to knowledge and beliefs experience gained at the time of schooling strongly influence the ways of teaching.

It is important, however, to think also about the teachers in practice and to organise further education and training for them in the form of seminars and workshops, where they could acquire the knowledge and skills urgently needed for the use of constructivism in teaching mathematics. In addition to the above teachers would certainly also gain trust in their own functioning, which is extremely important and necessary to successfully change one's ways of working with students.

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Abstract

Presented In the article is constructivism in the learning and teaching of mathematics. Constructivism is the belief students possess certain prior knowledge and experiences, on which their assumptions are based that serve as the foundation for the formation of the context or as the base for solving a concrete problem.

If we wish the constructivist model of learning and teaching to be transferred into school practice, it must also be introduced in the teaching of teachers-to-be at universities—at faculties

of education. This means teachers-to-be must themselves get acquainted with this kind of learning and teaching, as besides knowledge and beliefs also one's own experience gained at the time of schooling influences the way of teaching.

Key words: learning, teaching, constructivism, teaching mathematics